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ANALYTIC QCD RUNNING COUPLING WITH FINITE IR BEHAVIOUR AND UNIVERSAL $\overline{\alpha}_s(0)$ VALUE

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As is known fom QED, a possible solution to the ghost-pole trouble can be obtained by imposing the Q^2-analyticity imperative. Here, the pole is compensated by the α nonanalytic contribution that results in finite coupling renormalization.

We apply this idea to QCD and arrive at the Q^2 analytic $\overline{\alpha}_s(Q)$. This solution corresponds to perturbation expansion, obeys AF and, due to nonperturbative contribution, has a regular IR behaviour. It does not contain any adjustable parameter and has a finite IR limit $\overline{\alpha}_s(0)$ which depends only on group symmetry factors.

In the one-loop approximation it is equal to $4\pi/\beta_0 \simeq 1.40$ and turns out to be surprisingly stable with respect to higher order corrections. On the other hand, the IR behaviour of our new analytic solution agrees with recent global low energy experimental estimates of $\overline{\alpha}_s(Q^2)$.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Аналитическая бегущая константа связи КХД с конечным ИК поведением и универсальное значение $\overline{\alpha}_s(0)$

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Как известно из КЭД, возможное решение проблемы призрачного полюса может быть получено наложением условия аналитичности по Q^2. При этом полюс компенсируется выражением, неаналитичным по α, которое приводит также к конечной перенормировке заряда.

Мы используем эту идею для случая КХД и получаем для $\overline{\alpha}_s(Q)$ аналитическое выражение. Это решение обладает свойством асимптотической свободы, а его регулярность в инфракрасной области обязана непертурбативным вкладам. Оно не содержит дополнительных параметров и в инфракрасной области имеет конечный предел $\overline{\alpha}_s(0)$, зависящий лишь от симметрийных факторов.

В однопетлевом приближении он равен $\overline{\alpha}_s(0) = 4\pi/\beta_0 \simeq 1.40$ и оказывается удивительно стабильным по отношению к высшим поправкам (тем самым схемно-инвариантным). Полученное аналитическое решение также согласуется с недавними интегральными экспериментальными оценками поведения $\overline{\alpha}_s(Q^2)$ в инфракрасной области.

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1. We recall first some results obtained in QED about 40 years ago. The QED effective coupling $\bar{\alpha}_s(Q^2)$, being proportional to the transverse dressed photon propagator amplitude, is an analytic function in the cut complex Q^2 plane and satisfies the Källen-Lehmann spectral representation. The «analytization procedure» elaborated in papers [1,2] consists of three steps:

1) Find an explicit expression for $\bar{\alpha}_{RG}(Q^2)$ in the space-like region by a standard RG improvement of perturbative input. Continue this expression to the time-like Q^2 domain.

2) Calculate the imaginary part of $\bar{\alpha}_{RG}(-Q^2)$ on the cut and define the spectral density $\rho_{RG}(\sigma, \alpha) = \mathrm{Im}\,\bar{\alpha}_{RG}(-\sigma, \alpha)$.

3) Using the spectral representation with ρ_{RG} in the integrand, define an analytic $\bar{\alpha}_{an}(Q)$.

For one-loop massless QED, this procedure produces [2] an explicit expression (see Eq.(2.6) in Ref.[2] or analogous QCD Eq.(2) below) which has the following properties:

a) It has no ghost pole;

b) Considered as a function of α in the vicinity of the point $\alpha = 0$ it has an essential singularity of the $\exp(-3\pi/\alpha)$ type;

c) In the vicinity of this singularity for real and positive α it admits an asymptotic expansion that coincides with usual perturbation theory;

d) It has a finite ultraviolet asymptotic limit, $\bar{\alpha}(\infty, \alpha) = 3\pi$, which *does not depend on the experimentally input value* $\alpha \simeq 1/137$.

The same procedure in the two-loop massless QED approximation yielded [2] a more complicated expression with the same essential features.

2. To use the same technique in QCD one has to make two observations. First, since $\bar{\alpha}_s(Q^2)$ has to be defined via a product of propagators and a vertex function, validity of the spectral representation is not obvious. However, this validity has been established in Ref.[3] on the basis of analytic properties of the vertex diagrams. Second, for QCD with an arbitrary covariant gauge, the running of the coupling and gauge parameter are interconnected. For simplicity, we assume that the $\overline{MS}$ scheme is used.

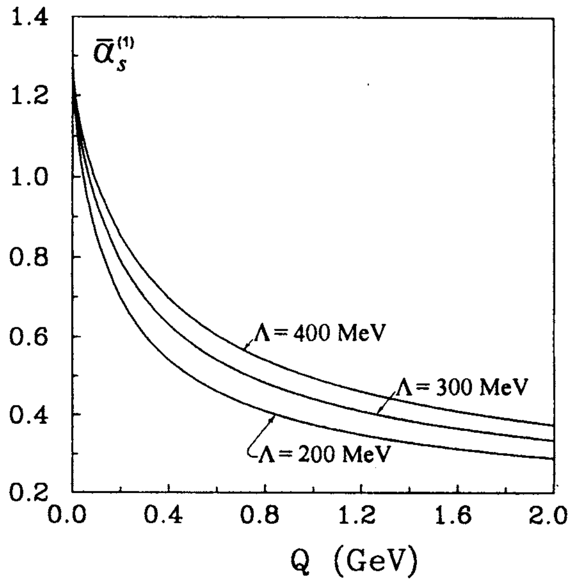
In the following we use the spectral representation in the nonsubtracted form

$$\bar{a}(Q^2) = \frac{\bar{\alpha}_s(Q^2)}{4\pi} = \frac{1}{\pi}\int_0^\infty \frac{\rho(\sigma, \Lambda)\, d\sigma}{\sigma + Q^2 - i\varepsilon}\,. \tag{1}$$

The usual massless one-loop RG approximation yields the spectral function

$$\rho^{(1)}_{RG}(\sigma, \Lambda) = \frac{\pi}{\beta_0(l^2 + \pi^2)}\,, \quad l = \ln\frac{\sigma}{\Lambda^2}\,, \quad \beta_0 = 11 - \frac{2}{3}n_f\,.$$

Substituting $\rho^{(1)}_{RG}$ into spectral representation Eq.(1) we get



The behaviour of the analytic running coupling constant $\overline{\alpha}_s^{(1)}$ Eq.(2) versus Q at different Λ values

$$\overline{a}_{an,s}^{(1)}(Q^2) = \frac{1}{\beta_0}\left[\frac{1}{\ln Q^2/\Lambda^2} + \frac{\Lambda^2}{\Lambda^2 - Q^2}\right], \tag{2}$$

where we have used the QCD scale parameter Λ. The «analytic» running coupling Eq.(2) has no ghost pole and its limiting value

$$\overline{\alpha}_s^{(1)}(0) = 4\pi/\beta_0 \tag{3}$$

does not depend on Λ being a *universal constant which depends only on group factors.*

We have become accustomed to the idea that theory supplies us with a family of possible curves for $\overline{\alpha}_s(Q^2)$ and one has to choose the «physical one» of them by comparing with experiment. Here, in Eq.(2) the whole bunch of possible curves for $\overline{\alpha}_s(Q^2)$ corresponding to different Λ have the same limit at $Q^2 = 0$ as shown on the Figure. For $n_f = 3$ it is equal to

$$\overline{\alpha}_s^{(1)}(0) = 4\pi/9 \simeq 1.396. \tag{4}$$

Another feature of Eq.(2) is the fact that its correct analytic properties in the IR domain are provided by a nonperturbative contribution* like $\exp(-1/a\beta_0)$.

*The connection between «Q^2 analyticity» and «α nonperturbativity» has been discussed in Ref.[4].

To investigate the stability of the result, Eq.(4), with respect to the next loop corrections, we have considered the two-loop approximation to $\bar{\alpha}_s(Q^2)$ in the form

$$\bar{\alpha}^{(2)}_{RG} = \frac{1}{\beta_0[L + b_1 \ln L]}, \quad L = \ln \frac{Q^2}{\Lambda^2}, \quad b_1 = \frac{\beta_1}{\beta_0^2}, \quad \beta_1 = 102 - \frac{38}{3} n_f$$

and

$$\rho^{(2)}_{RG}(\sigma, \Lambda) = \frac{I(l)}{\beta_0[R^2(l) + I^2(l)]}, \qquad l = \ln \frac{\sigma}{\Lambda^2},$$

$$R(l) = l + b_1 \ln (\sqrt{l^2 + \pi^2}), \quad I(l) = \pi + b_1 \arccos \frac{l}{\sqrt{l^2 + \pi^2}}. \tag{5}$$

The limiting two-loop value $\bar{\alpha}^{(2)}_s(0)$ is also specified only by group factors via β_0 and β_1. Surprisingly, this value found by numerical calculation practically coincides with the one-loop result. For the $\overline{MS}$ scheme in a three-loop approximation $\bar{\alpha}^{(3)}_s(0)$ changes by a few per cent. Thus, the value $\bar{\alpha}_s(0)$ is remarkably stable with respect to higher loop corrections and is practically independent of renormalization scheme.

3. To fix Λ we use the reference point $M_\tau = 1.78$ GeV with $\bar{\alpha}_s(M_\tau^2) = 0.33 \pm 0.03$ [5]. Corresponding solutions $\bar{\alpha}^{(l\,=\,1,2,3)}_s(Q^2)$ are very close to each other for the interval of interest $Q^2 \leq 10$ GeV^2. For instance, at $Q^2 = 10$ GeV^2 we have $\bar{\alpha}^{(1)}_s(10) \simeq \bar{\alpha}^{(2)}_s(10) =$ $= 0.267$, $\bar{\alpha}^{(3)}_s(10) = 0.265$. Here, again we used $n_f = 3$ as the average number of active quarks in the spectral density. This seems to be reasonable in the IR region.

For a more realistic description of the evolution of $\bar{\alpha}_s(Q^2)$ in the Euclidean region 3 GeV $< Q <$ 100 GeV, one should take into account heavy quark thresholds. Using the explicitly mass-dependent RG formalism [7] developed in the 50's, a «smooth matching» algorithm has been devised recently [6]. This can be given to correct analytic properties while incorporating heavy quark thresholds.

The idea that $\bar{\alpha}_s(Q^2)$ can be frozen at small momentum has been recently discussed in some papers (see, e.g., Ref.[8]). There seems to be experimental evidence indicating behaviour of this type for the QCD coupling. As the appropriate object for comparison with our theoretical construction we use the average

$$A(k) = \frac{1}{k} \int_0^k dQ \bar{\alpha}_s(Q^2, \Lambda). \tag{6}$$

«Experimental» estimates for this integral are $A(2\ \text{GeV}) = 0.52 \pm 0.10$ GeV [9] and $A(2\ \text{GeV}) = 0.57 \pm 0.10$ GeV [10]. Our one-loop results in case Eq.(2) for A are summarized below for a few reference values of $\bar{\alpha}_s(M_\tau^2)$.

$\bar{\alpha}_s(M_\tau^2)$	0.34	0.36	0.38
$A(2\ \text{GeV})$	0.50	0.52	0.55

Note here that a nonperturbative contribution, like the second term in l.h.s. of Eq.(2), reveals itself even at moderate Q values by «slowing down» the velocity of the $\bar{\alpha}_s(Q^2)$ evolution. For instance, in the vicinity of c and b quark thresholds at $Q = 3$ GeV it contributes about 4% which could be essential for the resolution of the «discrepancy» between DIS and Z_0 data for $\bar{\alpha}_s(Q^2)$.

4. We have argued that a regular analytic behaviour for $\bar{\alpha}_s(Q^2)$ in the IR region could be provided by nonperturbative contributions which can be considered as a sum of powers of Λ^2/Q^2.

Probably, our most curious result is the stability of a «long-range intensity of strong interaction», $\bar{\alpha}_s(0)$, as well as the $\bar{\alpha}_s(Q^2)$ IR behaviour that turns out to agree reasonably well with experimental estimates.

On the other hand, the nonzero $\bar{\alpha}_s(0)$ value evidently contradicts the confinement property. To satisfy this, one should have

$$\bar{\alpha}_s(0) = 0$$

as it has recently been emphasized by Nishijima [11] in the context of the connection between asymptotic freedom (AF) and color confinement (CC).

It is possible to correlate this type of the IR limiting behaviour with the RG-improved perturbative input and Q^2-analyticity by inserting a Castillejo–Dalitz–Dyson zero into our solution (see paper [12]). Such a generalyzed analytic solution will contain additional parameters. In this construction there is no evident relation between AF and CC. Here, CC is provided by nonperturbative contributions.

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References

1. Redmond P. — Phys. Rev., 1958, v.112, p.1404.
2. Bogoliubov N.N., Logunov A.A., Shirkov D.V. — Sov. Phys. JETP, 1959, v.37, p.805.
3. Ginzburg I.F., Shirkov D.V. — Sov. Phys. JETP, 1965, v.49, p.335.
4. Shirkov D.V. — Lett. Math. Phys., 1976, v.1, p.179.

5. Narison S. — Phys. Lett., 1995, v.B361, p.121.
6. Dokshitzer Yu.L., Shirkov D.V. — Zeit. f. Phys., 1995, v.67, p.449;
Shirkov D.V., Mikhailov S.V. — Zeit. f. Phys., 1994, v.63, p.463.
7. Bogoliubov N.N., Shirkov D.V. — Nuovo Cim., 1956, v.3, p.845.
8. Mattingly A.C., Stevenson P.M. — Phys. Rev., 1994, v.D49, p.437.
9. Dokshitzer Yu.L., Webber B.R. — Phys. Lett., 1995, v.B352, p.451.
10. Dokshitzer Yu.L., Khoze V.A., Troyan S.I. — Phys. Rev., 1996, v.D53, p.89.
11. Nishijima K. — Czech. J. Phys., 1996, v.46, p.1.
12. Kirzjnits D.A., Fainberg V.Ja., Fradkin E.S. — Sov. Phys. JETP, 1960, v.38, p.239.

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